

A proposal for static load tests on piles: the Equilibrium Method

Francisco R. Lopes^{1#} , Paulo Eduardo L. Santa Maria¹ , Fernando A. B. Danziger¹ ,
Ian S. M. Martins¹ , Bernadete R. Danziger² , Michel C. Tassi³ 

Technical Note

Keywords

Pile load tests
Equilibrium Method
Pile foundations
Pile bearing capacity
Soil viscosity

Abstract

This note presents an alternative method for static load tests on piles (and caissons). Called Equilibrium Method by its first proponents, the method was applied in some load tests in Brazil, in addition to being the object of theoretical studies conducted at the Federal University of Rio de Janeiro. The method consists, in each step, to keep the load constant for a period of time and then let it relax (not pumping the jack) until the displacement and the load reach mutual equilibrium. The stabilized displacement and the relaxed load (the so-called *load and displacement in equilibrium*) are considered for the load-displacement curve. The method has the advantage of producing the load-displacement curve close to that of a slow, stabilized test (incremental slow maintained load test), but with a shorter total execution time. The paper includes a short theoretical background and a review of the Brazilian experience.

1. Introduction

Soils, most notably clayey, saturated, exhibit viscous behaviour, that is, a time-dependent behaviour which is not associated with water migration to equilibrate pore-pressures – *consolidation* –. Viscosity manifests itself in some conditions, such as *creep* (deformation under constant loading conditions), *stress relaxation* (change in stress under sustained displacement) and the *effect of loading rate* on shear strength. These occurrences or phenomena were recognized a long time ago, as in the work of Buisman (1936), who described what became known as *secondary consolidation* (which would be *creep*), and in those of Casagrande & Wilson (1951) and Bjerrum (1973), in which a variation in the shear strength of clays was observed with the variation of *loading rate*. Viscosity is also responsible for increasing the thrust on retaining structures, evolving to an *at-rest condition*, if these are prevented from displacing (e.g., Bishop, 1957). Early works, such as Hvorslev (1937, 1960) and Terzaghi (1941), attributed these phenomena to the viscous nature of the adsorbed water film involving soil particles.

1.1 An approach to soil viscosity and its effects on pile capacity

A viscosity model developed at the Graduate School of Engineering, Federal University of Rio de Janeiro assumes

that the shear stress in clayey soils has two components: one of frictional nature and the other of viscous nature, i.e. (Martins, 1992):

$$\tau = \tau_f + \tau_\eta \quad (1)$$

It turns out that the frictional component of the shear stress depends on the effective solid-solid stress, σ'_s , and on the mobilized friction angle, ϕ'_{mob} , with the mobilized friction angle being, in turn, a function of the distortion, γ . Thus, the friction component is written:

$$\tau_f = \sigma'_s \tan \phi'_{mob}(\gamma) \quad (2)$$

On the other hand, the viscous component, τ_η , is a function of a soil viscosity coefficient, η (e), – in its turn a function of the void ratio, e , – and of the rate of distortion, $d\gamma/dt$. Thus, the viscous component of the shear stress can be written as:

$$\tau_\eta = \eta(e) \frac{d\gamma}{dt} \quad (3)$$

Therefore, the shear stresses mobilized, during pile loading, both along the shaft and in the soil region that produces base or tip resistance, can be expressed by the sum of τ_f and τ_η .

[#]Corresponding author. E-mail address: flopes@coc.ufirj.br

¹Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, Brasil.

²Universidade do Estado do Rio de Janeiro, Rio de Janeiro, RJ, Brasil.

³Instituto Militar de Engenharia, Exército Brasileiro, Rio de Janeiro, RJ, Brasil.

Submitted on September 23, 2020; Final Acceptance on January 20, 2021; Discussion open until May 31, 2021.

DOI: <https://doi.org/10.28927/SR.2021.057920>



This is an Open Access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This means that the expression for the mobilized shaft load capacity must be written, taking into account the rate effect (viscosity), as:

$$Q_s = U \int_0^L K \cdot \sigma_v'(z) \cdot \tan \phi_{mob}' \cdot dz + U \int_0^L \eta(e(z)) \frac{d\gamma}{dt} dz \quad (4)$$

The friction angle, as it is usually determined, is also affected by rate effects, and, therefore, the tip resistance is also a function of the loading rate in the load test, that is, the higher the loading rate – or the shorter the time taken to produce failure –, the greater the tip resistance. In relation to the pile tip, there is the opposite time effect due to the consolidation (dissipation of excess pore-pressures generated by soil compression under the tip). But it can be said that the total load capacity (shaft + tip capacities), measured in a load test, increases with loading rate.

2. Loading rate effects on pile bearing capacity

The behaviour observed in pile load tests is typical of loading rate effects on soil resistance, that is, the faster a pile is loaded – or the shorter the duration of load stages – the greater the resistance. This behaviour of piles, in which quick loadings bring about higher capacities than slow loadings, is opposed to that of plates on saturated clayey soils in

which fast loading tends to be critical. This is explained by the stress-paths followed at representative points around these foundations (Figure 1). Under a plate, stress paths are close to that of a triaxial test, in which there is an increase in mean normal stress accompanying the increase in shear stress (Figure 1b); thus, there is an increase in pore-pressures with loading, which – if dissipated in a slow loading process – lead to higher resistance. On the other hand, in the soil around the pile shaft, the stress path is vertical, indicating a loading mode called *simple shear* (Figure 1a); thus, unless the soil is contractive, there will be practically no excess pore-pressure generation during loading. If there is no water migration process (consolidation) in this region, it can be concluded that viscosity dominates the time dependent behaviour of the soil around the pile shaft. Under the tip of the pile, the stress path is similar to that of the soil under plates, but this resistance is only a fraction of the total pile resistance (unlike the plate). In other words, in piles, which are long elements, there is a large portion of soil subject to an increase in resistance with an increase in rate, therefore, there is a predominance of viscous effects over consolidation.

The assumption that quick loading leads to lower load capacity – which is only valid for plates – served to postulate the load test known in Brazil as the *mixed method*. In this method, loading up to the service load follows a

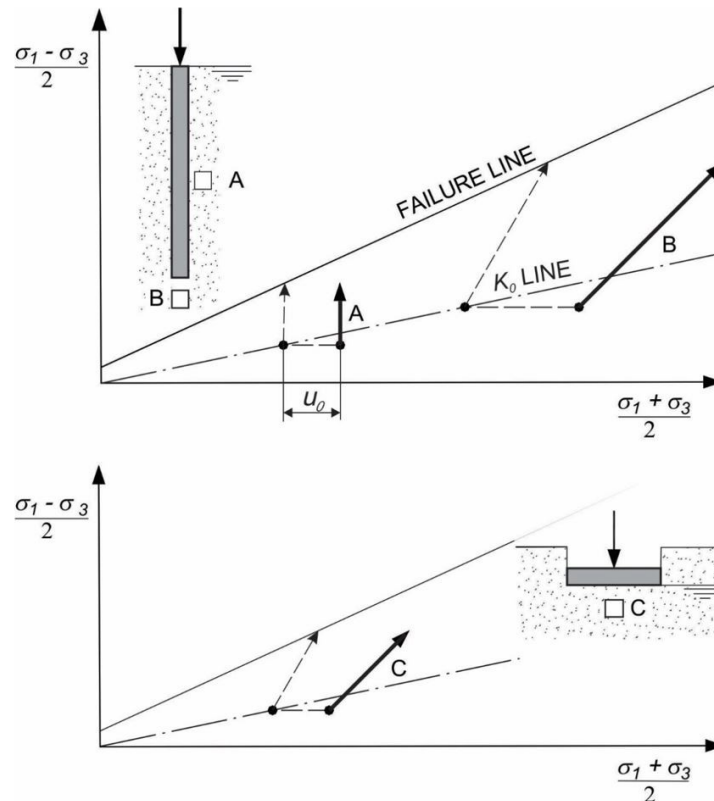


Figure 1. Total (solid lines) and effective stress paths (dashed lines) at points (a) around a pile and (b) under a plate, during loading (adapted from Lopes, 1979, 1985).

stabilization criterion (i.e., in a slow loading rate) in order to determine the displacement for the service load, and then it proceeds with short duration load increments (i.e., in a quick loading rate), assuming that the ultimate load capacity obtained is on the safe side (lower than that under slow loading).

Pore-pressure generation around the pile shaft during load tests should not be mistaken for pore-pressure generation when installing driven piles, which is significant. The dissipation of installation pore-pressures is the main cause of the gain in pile capacity with time after installation, known as *set-up*.

The issue of quick tests, an example being the CRP (Constant Rate of Penetration) test, indicating load capacities greater than slow tests, was discussed by several authors, such as Whitaker & Cooke (1966), Lopes (1985, 1989), Ferreira & Lopes (1985), Burland & Twine (1988), Patel (1992) and England & Fleming (1994). These latter authors stated:

It has been shown that the effect of the rate of penetration (normally approximately 1 mm/min) is to enhance pile shaft capacities in clay soils, but the same is also probable with regard to friction in a wider range of soils and also to base capacities.

2.1 Loading rate effect on displacements

In relation to the displacement for service loads, there is no doubt that a slow, stabilized load is that representative of a foundation – plate or pile – under maintained load.

2.2 Methods for obtaining a stabilized load-displacement curve

The *fully stabilized* load-displacement curve corresponds to the *zero loading rate curve*. The question is how to arrive at this curve in load tests, in which, invariably, the load is applied in stages. There are two ways (see Figure 2): (i) applying a load increment and keeping it constant until displacements cease (path A-B) or (ii) applying an increase in load and allowing both displacements and loads to stabilize (path A-C). In option (ii), stabilization will imply load relaxation. The study of the Appendix shows that the path via relaxation is faster.

Experience shows that the time for stabilization under constant load increases as the loading level increases. At the higher load stages, several hours are required for rigorous stabilization. In the Equilibrium Method, according to Mohan et al. (1967), stabilization is achieved ‘in a matter of minutes’.

3. The Equilibrium Method

The Equilibrium Method consists, in each step, of keeping the load constant for a period of time and then let it relax (not pumping the jack) until the displacement and the load reach mutual equilibrium. The stabilized displacement and the relaxed load (the so-called *load and displacement in equilibrium*) are considered for the load-displacement curve. The set of graphs produced by the method is shown

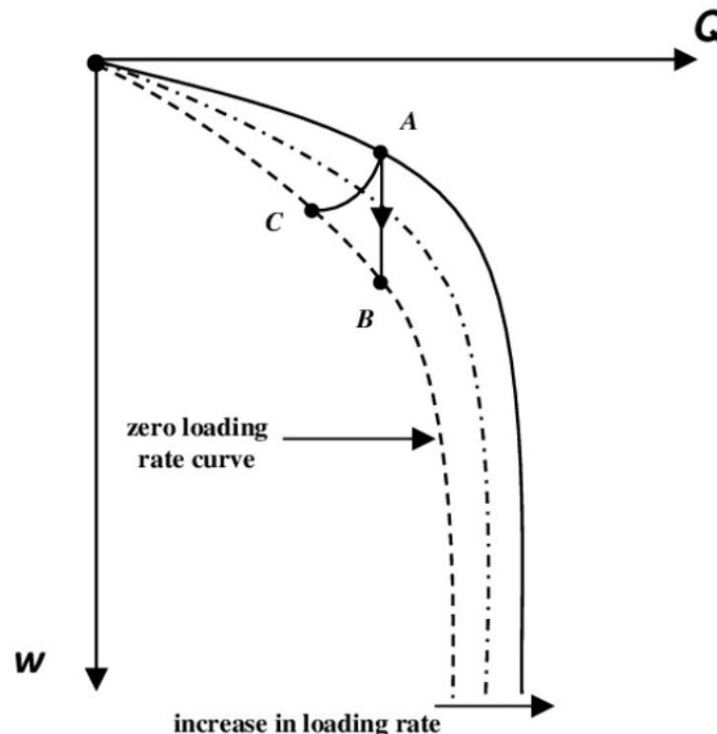


Figure 2. Possible load (Q) vs. displacement (w) curves in load tests and paths to reach the zero loading rate curve (Martins, 2006).

in Figure 3, where t_1 is the time interval under constant load and t_2 is the time interval of load relaxation.

4. Brazilian experience with the Equilibrium Method

4.1 Load tests at Santos-São Vicente Bridge (DERSA)

Ferreira (1985) analyzed 6 load tests carried out on 2 steel pipe piles of a bridge between Santos and São Vicente. The piles were 65 cm in diameter and 42 and 50 m long. Soil profile was a sequence of layers of soft clay and low density clayey fine sand, until nearly 40m, where residual soil was found (dense sandy silt).

The service load of the piles was 2500 kN and the maximum loads in the tests reached 6000 kN. Three procedures, applied in sequence, were followed for each of the piles:

- (i) incremental load until a rigorous stabilization, maximum load of 5000 kN, in 10 stages of 16 hours each;
- (ii) Brazilian standard NBR 6121 (ABNT, 1980), maximum load of 3750 kN, in 8 stages;
- (iii) Equilibrium Method, maximum load of 6000 kN, in 10 stages.

The load tests lasted about 50 total hours in the last two procedures and about 200 hours in the more rigorous stabilization procedure.

In terms of displacements, for the 3000 kN stage (the closest to the service load, 2500 kN), displacements were small and close in the 3 methods: 8mm for PV-02 and 6mm for PV-03. These displacements reflect the fact that the piles had their tips driven into very dense material, which was also reflected in the small load relaxation in the stages of the Equilibrium Method.

Five load-displacement curves did not indicate a clear failure and extrapolations by Van der Veen's (1953) method indicated unrealistic load capacities, around 9000 kN. Only the Equilibrium Method curve of PV-02 showed 80 mm displacement for the maximum load, indicating failure for practical purposes (~ 6000 kN). Figure 4 shows the results of PV-02 for the more rigorous stabilization procedure and for the Equilibrium Method (with the rigorous stabilization curve extrapolated).

4.2 Load test on model pile in soft clay in Rio de Janeiro

Francisco (2004) performed load tests on a steel model pile, 11.5 cm diameter, driven in soft clay to a depth of 3.5 m, at Sarapuí II test site, Rio de Janeiro metropolitan region. The pile was subject to a quick load test and to 2 equilibrium tests (Figure 5). Failure loads were in the proximity of 7.2 kN for the quick test and between 5.5 and 6.5 kN for the equilibrium tests. The test program also included a long term creep test and the thesis presents a theoretical approach to pile behavior considering soil viscosity.

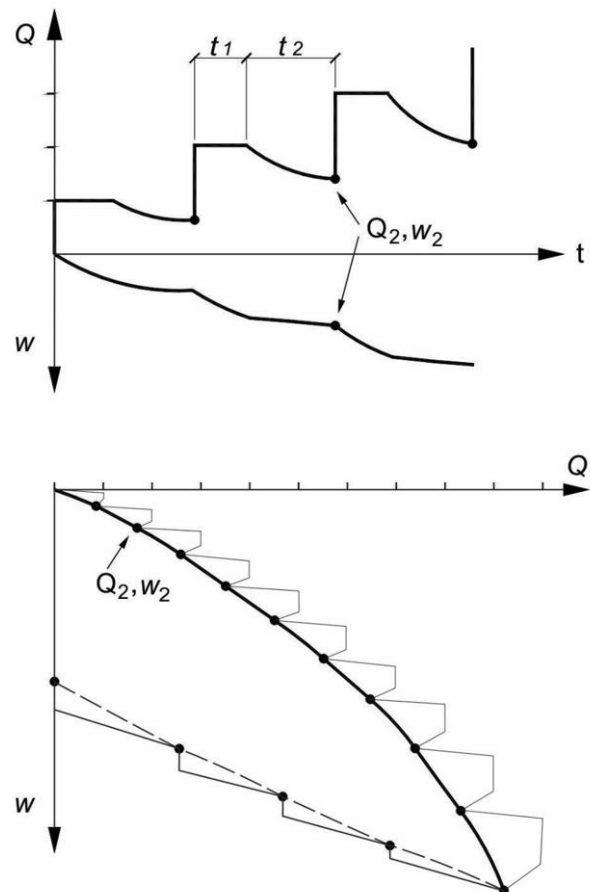


Figure 3. Typical graphs produced in the application of the Equilibrium Method.

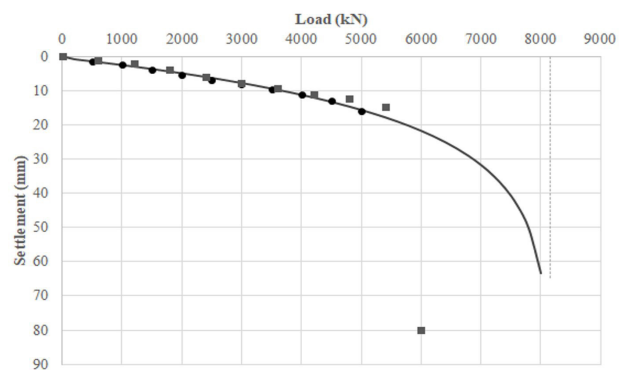


Figure 4. Load-displacement curves obtained with incremental load maintained until rigorous stabilization (circles) - with extrapolation - and by the Equilibrium Method (squares), PV-02 (Ferreira, 1985).

4.3 Load tests at USP/São Carlos test site

Benvenuti (2001) performed load tests on two caissons, 50 cm shaft diameter, 1.5 m base diameter, length 5.1 m, installed in collapsible soil at the São Carlos Test Site, State of São Paulo. On each caisson, 4 tests were performed: 3 quick and then 1 by the Equilibrium Method. One caisson was tested in natural water content conditions and the other after

flooding. Results for the natural water content conditions are shown in Figure 6. It can be seen that the Equilibrium Method produced load-displacement curves with less stiffness in the first-loading segment.

4.4 Load tests on model plates at University of São Paulo/São Carlos

Almeida (2009) carried out load tests – although on plates – in the laboratory, on undisturbed block-type samples of partially saturated soils. Sets of three samples were taken next to each other, presenting the same matrix suction. On each sample one of the following test methods was applied: (i) slow maintained load, (ii) quick maintained load and (iii) Equilibrium Method (with 5 minutes of maintained load and 10 minutes of load relaxation). It was concluded that the load-displacement curves obtained with the Equilibrium Method were closer to those of the slow maintained load tests than those of the quick tests, as shown in Figure 7 for one of the test sets.

5. Proposed procedure for the Equilibrium Method

Based on the published data on the Equilibrium Method, the following procedure is proposed (see Figure 3):

- loading must be carried out in 10 equal stages, each one corresponding to 20% of the expected service load;
- the load of each stage must be kept constant for 20 min, taking displacement readings at 2, 5, 10, 15 and 20 min;
- after 20 min, the load is allowed to relax for a period of 15 min, noting displacements and loads at 2, 5, 10, 15 min;
- at 15 min, stabilization is verified by the criterion described below; if the criterion is met, the stage ends; if not, the stage continues up to a maximum of 30 min, checking the stabilization criteria at 20 and 25 min;
- the load and displacement after relaxation will be considered for the load-displacement curve;
- unloading may be carried out in 4 short duration stages, such as 10 min each one.

5.1 Criterion to end the relaxation period

During relaxation, the load variation (ΔQ in Figure 8) is more pronounced than the displacement variation (Δw_2), therefore, the stabilization criterion is applied to the former. The proposed criterion compares the load variation that occurred up to a given time to the variation in the previous time (see Figure 8). If the ratio between the 2 load variations is less than 5%, the stage is terminated. The application of this criterion starts at 15 min. Therefore, if

$$\Delta Q_{15} \leq 1,05 \Delta Q_{10} \quad (5)$$

the stage ends; if not, go on to 20 min, and so on for up to 30 min (maximum relaxation time).

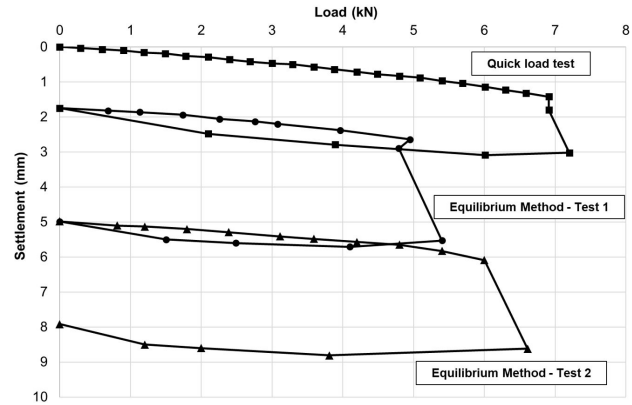


Figure 5. Load-displacement curves for a model pile subjected to different loading procedures (Francisco, 2004).

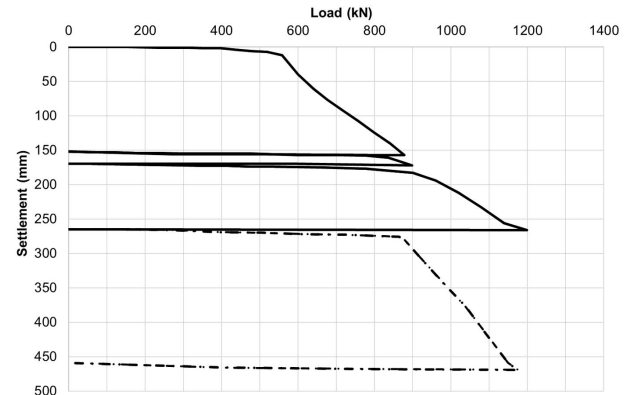


Figure 6. Load-displacement curves from quick tests (solid line) and Equilibrium Method (dash-dotted line) in natural water content conditions (Benvenuti, 2001).

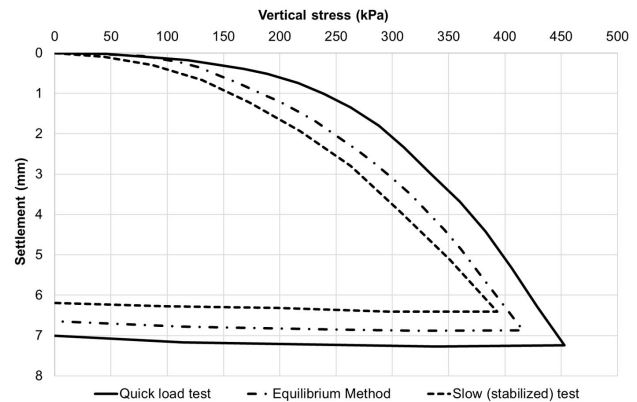


Figure 7. Pressure-displacement curves from plate load tests: quick test, slow test and Equilibrium Method (Almeida, 2009).

For the interpretation of the ultimate pile capacity, any procedure from the local Foundation Code or established in the literature can be applied, as in any other type of test method.

This procedure should be evaluated with the experience gathered with new load tests.

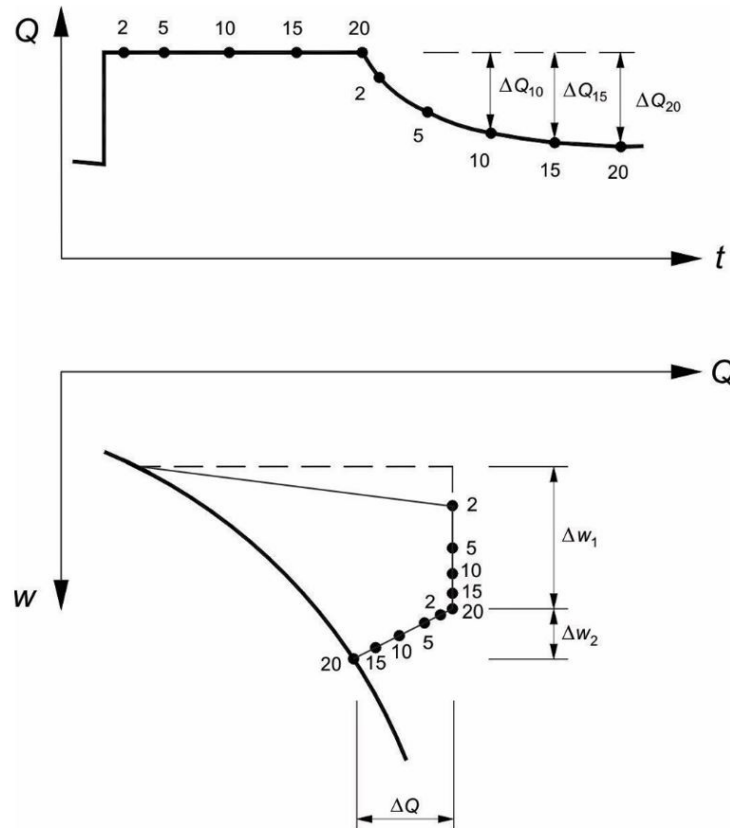


Figure 8. Details of a loading stage – Equilibrium Method – with reading times (in min) indicated.

6. Concluding remarks

This note aims to stimulate the discussion of procedures for carrying out load tests on piles and caissons. There is an interest in limiting the time spent on load tests, for various reasons, such as interference in the construction schedule, labor safety and costs. The method discussed here, called the Equilibrium Method by its first proponents (Mohan et al., 1967), can lead to displacements very close to those of a fully stabilized test in a predictable execution time.

It is noteworthy that the choice of the load test method must be made by the Designer and/or Consultant, taking into account the particularities of the load to which the pile/caisson will be subjected under the structure. Among the methods is the quick test, which should not be understood as a static load test, but a test that reflects the behaviour of the pile under fast acting loads, such as wind and wave actions on power transmission towers and marine structures.

Acknowledgements

The authors benefited from discussions with members of the review committee of the Brazilian Load Test Standard, in particular Paulo J.R. Albuquerque, Guilherme Soler and Gentil Miranda Jr. Extrapolations by Van der Veen's method made use of a spreadsheet by Prof. José Antonio Schiavon, Technological Institute of Aeronautics. This study was partly

financed by CAPES - Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, Brazil, under Finance Code 001.

Declaration of interest

There is no conflict of interests in the material presented.

Author's contributions

Francisco R. Lopes: conceptualization, methodology, writing - original draft preparation. Paulo Eduardo L. Santa Maria: conceptualization, formal analysis. Fernando A.B. Danziger: investigation, discussion of results, review and approval of the final version of the manuscript. Ian S. M. Martins: conceptualization, investigation. Bernadete R. Danziger: discussion of results, writing - reviewing and editing. Michel C. Tassi: visualization, discussion of results, review and approval of the final version of the manuscript.

References

- ABNT NBR 6121 (1980). NBR 6121: *Pile and Caisson - Load test*. ABNT - Associação Brasileira de Normas Técnicas, Rio de Janeiro, RJ (in Portuguese).
- Almeida, M.P.B. (2009). *Quick load test with equilibrium settlement* [Unpublished master's dissertation]. Escola de Engenharia de São Carlos, Universidade de São Paulo (in Portuguese).

- Benvenuti, M. (2001). *Caissons to improve load capacity on collapsible soils* [Unpublished master's dissertation]. Escola de Engenharia de São Carlos, Universidade de São Paulo (in Portuguese).
- Bishop, A.W. (1957). Discussion, Session 8 - Earth Pressure on Structures and Tunnels. In *Proceedings of the 4th International Conference on Soil Mechanics and Foundation Engineering* (pp. 242-243). London: ISSMGE - International Society for Soil Mechanics.
- Bjerrum, L. (1973). Problems of soil mechanics and construction on soft clays and structurally unstable soils. In *Proceedings of the 8th International Conference on Soil Mechanics and Foundation Engineering* (pp. 111-189). Moscow: ISSMGE - International Society for Soil Mechanics.
- Buisman, A.S.K. (1936). Results of long duration settlement tests. In *Proceedings of the 1st International Conference on Soil Mechanics and Foundation Engineering* (Vol. 1, pp. 103-106). Cambridge, Mass: ISSMGE - International Society for Soil Mechanics.
- Burland, J.B., & Twine, D. (1988). The shaft friction of bored piles in terms of effective strength. In *Proceedings of the Seminar on Deep foundations on bored and auger piles* (pp. 411-420). Ghent: Balkema, Rotterdam .
- Casagrande, A., & Wilson, S. (1951). Effect of rate of loading on the strength of clays and shales at constant water content. *Geotechnique*, 2(3), 251-263. <http://dx.doi.org/10.1680/geot.1951.2.3.251>.
- England, M., & Fleming, W.G.K. (1994). Review of foundation testing methods and procedures. *Proceedings of ICE. Geotechnical Engineering*, 107(3), 135-142. <http://dx.doi.org/10.1680/igeng.1994.26466>.
- Ferreira, A.C. (1985). *Loading rate effects and the question of pile settlements in load tests* [Unpublished master's dissertation]. Universidade Federal do Rio de Janeiro (in Portuguese).
- Ferreira, A.C., & Lopes, F.R. (1985). Contribution to the study of loading rate effects on the behaviour of test piles. *Proceedings of the 1st Seminar on Special Foundations*. São Paulo: ABEF - Associação Brasileira de Empresas de Engenharia de Fundações.
- Francisco, G.M. (2004). *Study of time effects on foundation piles in clayey soils* [Unpublished DSc Thesis]. Universidade Federal do Rio de Janeiro.
- Hvorslev, M.J. (1937). *Physical properties of remolded cohesive soils (Über die festigkeitseigenschaften gestörter bindiger boden)* [Unpublished Doctoral thesis]. U. S. Army Engineer Waterways Experiment Station.
- Hvorslev, M.J. (1960). Physical components of the shear strength of saturated clays. In *Proceedings of the ASCE Research Conference on Shear Strength of Cohesive Soils* (pp. 169-273). Boulder: University of Colorado.
- Lopes, F.R. (1979). *The undrained bearing capacity of piles and plates studied by the finite element method* [Unpublished PhD thesis]. Imperial College, University of London.
- Lopes, F.R. (1985). Lateral resistance of piles in clay and possible effect of loading rate. In *Proceedings of the Symposium on Theory and Practice of Deep Foundations* (Vol. 1, pp. 53-68). Porto Alegre: ABMS - Associação Brasileira de Mecânica dos Solos e Engenharia Geotécnica.
- Lopes, F.R. (1989). Discussion to Session 15. In *Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering* (Vol. 5, pp. 2981-2983). Rio de Janeiro: ISSMGE - International Society for Soil Mechanics.
- Martins, I.S.M. (1992). *Fundamentals of a model for the behaviour of saturated clayey soils* [Unpublished Doctoral thesis]. Universidade Federal do Rio de Janeiro (in Portuguese).
- Martins, I.S.M. (2006). On the viscous behaviour of soils. Lecture. In *Proceedings of the 13th Brazilian Conference Soil Mech.* Curitiba: ABMS - Associação Brasileira de Mecânica dos Solos e Engenharia Geotécnica.
- Mohan, D., Jain, G.S., & Jain, M.P. (1967). A new approach to load tests. *Geotechnique*, 17(3), 274-283. <http://dx.doi.org/10.1680/geot.1967.17.3.274>.
- Patel, D.C. (1992). Interpretation of results of piles tests in London Clay. In *Piling: European Practice and Worldwide Trends*. London: Thomas Telford.
- Terzaghi, K. (1941). Undisturbed clay samples and undisturbed clays. *Journal of the Boston Society of Civil Engineers*, 28(3), 211-231.
- Van der Veen, C. (1953). The bearing capacity of a pile. In: *Proceedings of the 3rd International Conference on Soil Mechanics and Foundation Engineering* (vol. 2, pp. 84-90). Zurich: ISSMGE - International Society for Soil Mechanics.
- Whitaker, T., & Cooke, R.W. (1966). An investigation of the shaft and base resistances of large bored piles in London Clay. In *Proceedings of the Large Bored Piles* (pp. 7-49). London: Institution of Civil Engineers.

Appendix. Comparison of evolution towards stabilization under maintained load and by load relaxation by Linear Viscoelasticity Theory

This appendix presents a comparative analysis of static load tests using the maintained load method and Equilibrium Method.

The resistance of a pile when subjected to generic external loads can be represented by the equation:

$$R = f1(w) + f2(\dot{w}) + f3(\ddot{w}) \quad (A1)$$

where $f1$, among other variables, is a function of the displacement w , $f2$ of the displacement rate (or velocity) \dot{w} and $f3$ of the acceleration \ddot{w} . In the case of static load tests, $f3(\ddot{w})$ can be neglected but not $f2(\dot{w})$. Although the second component is, in general, much less representative

than $f_l(w)$, it is important for understanding the process and its interpretation.

In order to better understand the displacement vs time behaviour and load vs time behaviour of piles during load test stages in the maintained load method and in the Equilibrium Method, a simple mathematical model of these tests was elaborated considering:

The use of Linear Viscoelasticity Theory;

The soil represented by the constitutive relations of Kelvin's viscoelastic model;

The test reaction structure represented by the linear elastic model.

Two important aspects should also be highlighted:

It is a very simple model and, therefore, the absolute values obtained are not relevant;

The objective is to compare the two load pile test procedures in terms of time to stabilize each process.

Figure A1 shows displacement development in a maintained load test in a 1000 kN loading stage. Figures A2 to A4 show the variation of load with time in a test by the Equilibrium

Method, considering different stiffnesses of the reaction system (1K, 2K, ..., 5K). In Figure A2, a displacement was applied such that the load value after stabilization was approximately equal to the loading stage of the maintained load test (~ 1000 kN). Figure A3 is the same as Figure A2, with an amplified time scale. In Figure A4, displacements were applied such that the initial load value was equal (for all stiffnesses of the reaction system) to the load in the maintained load test (1000 kN).

Appendix conclusions

1. It was observed that the time required for load stabilization in the load test by the Equilibrium Method varied relatively little when the stiffness of the reaction system varied from 1K to 5K.
2. The displacement stabilization time in an incremental maintained load test was approximately equal to three times the load stabilization time in the Equilibrium Method.

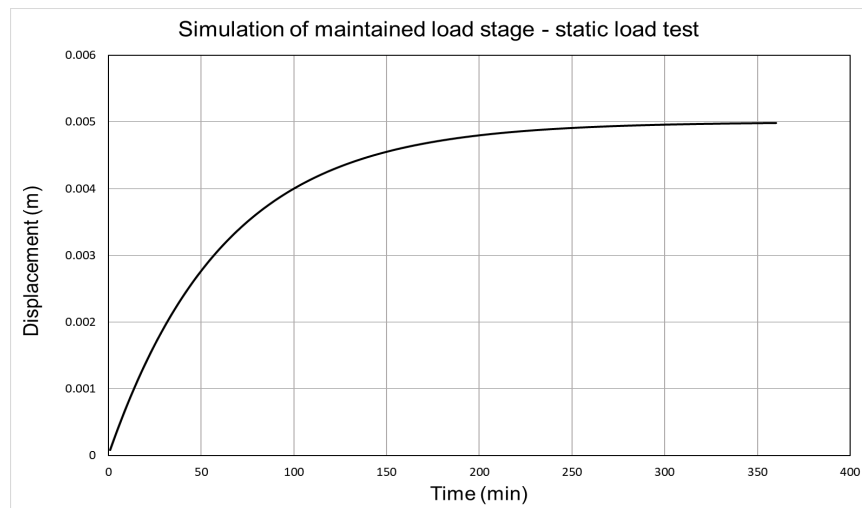


Figure A1. Simulation of (incremental) maintained load stage, conventional static load test.

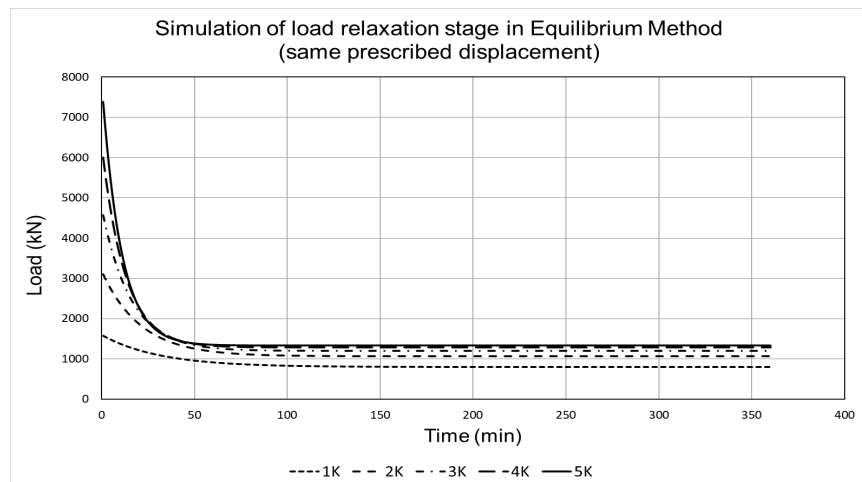


Figure A2. Simulation of load relaxation stage in Equilibrium Method, same prescribed displacement.

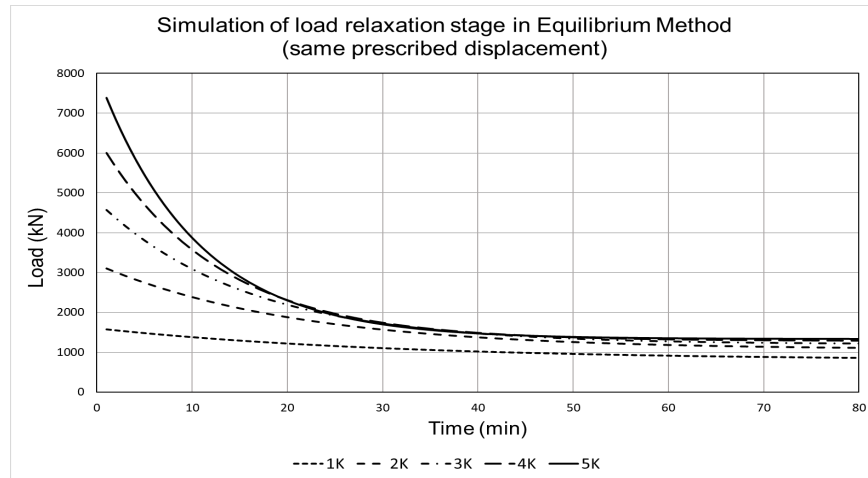


Figure A3. Simulation of load relaxation stage in Equilibrium Method, same prescribed displacement (first 80 minutes).

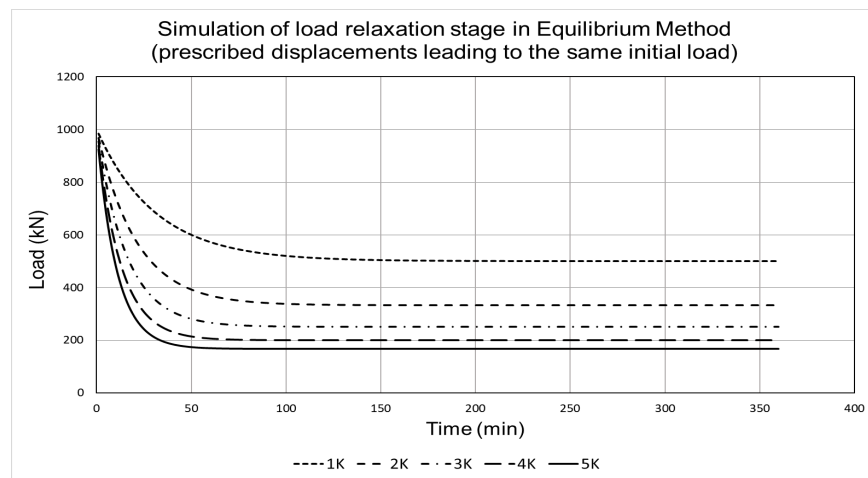


Figure A4. Simulation of load relaxation stage in Equilibrium Method, displacements leading to the same initial load.

List of Symbols

Q = load
 w = displacement
 Q_s = pile shaft load capacity
 U = pile perimeter
 L = pile length
 z = depth below ground level
 u = pore-pressure
 K = earth pressure coefficient after pile installation
 K_o = coefficient of earth pressure at rest
 e = void ratio
 t = time
 t_1 = time under constant load
 t_2 = time of load relaxation
 ΔQ = load variation in a stage
 Δw = displacement variation in a stage
 R = pile resistance
 $f1$ = displacement dependent factor
 $f2$ = rate (or velocity) dependent factor
 $f3$ = acceleration dependent factor
 \dot{w} = velocity
 \ddot{w} = acceleration
 ϕ'_{mob} = mobilized angle of shearing resistance
 τ = shear stress
 τ_f = friction component of shear stress
 τ_η = viscous component of shear stress
 σ'_s = effective solid-solid stress
 σ'_v = effective vertical stress
 γ = shear strain or distortion
 η = soil viscosity coefficient
 σ_I = major principal stress
 σ_3 = minor principal stress